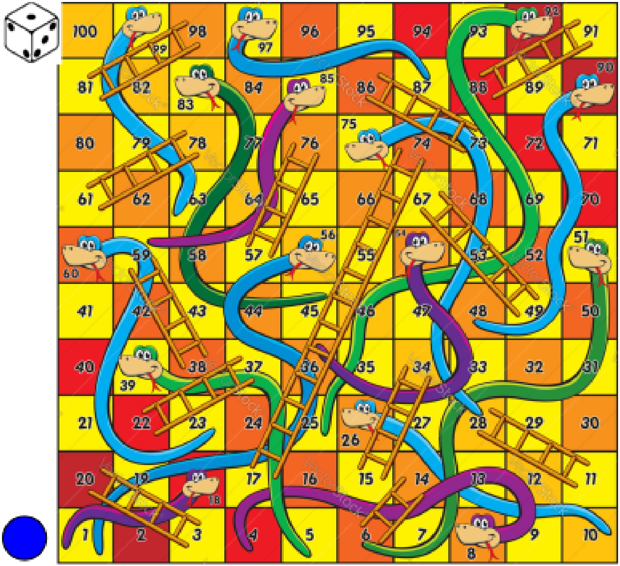
Probability distributions: Who cares & why?

Arnab Chakraborty

Indian Statistical Institute

Nov 12, 2017

“Snakes and Ladders” ludo



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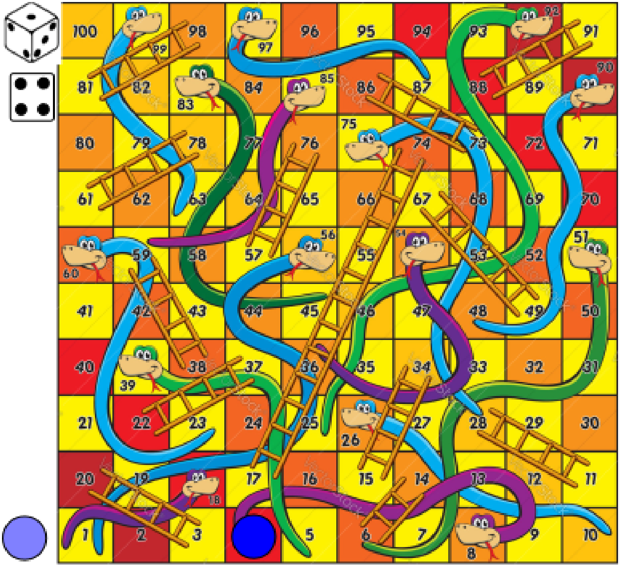

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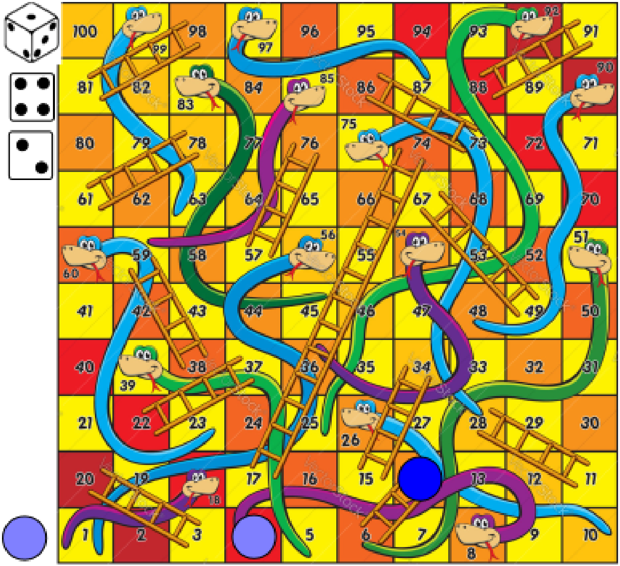


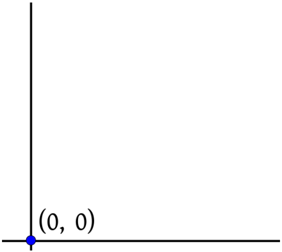
A strange ludo

1. $$\boxed{\begin{aligned} X_{new} &= 0.8X_{old} + 0.1 \\ Y_{new} &= 0.8Y_{old} + 0.04 \end{aligned}}$$

2. $$\boxed{\begin{aligned} X_{new} &= 0.5X_{old} + 0.25 \\ Y_{new} &= 0.5Y_{old} + 0.4 \end{aligned}}$$

3. $$\boxed{\begin{aligned} X_{new} &= 0.355X_{old} - 0.355Y_{old} + 0.266 \\ Y_{new} &= 0.355X_{old} + 0.355Y_{old} + 0.078 \end{aligned}}$$

4. $$\boxed{\begin{aligned} X_{new} &= 0.355X_{old} + 0.355Y_{old} + 0.378 \\ Y_{new} &= -0.355X_{old} + 0.355Y_{old} + 0.434 \end{aligned}}$$



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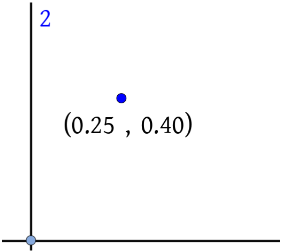

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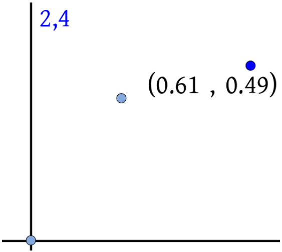

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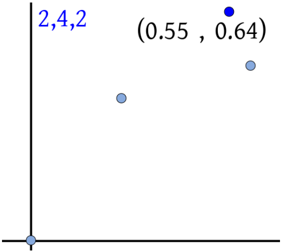

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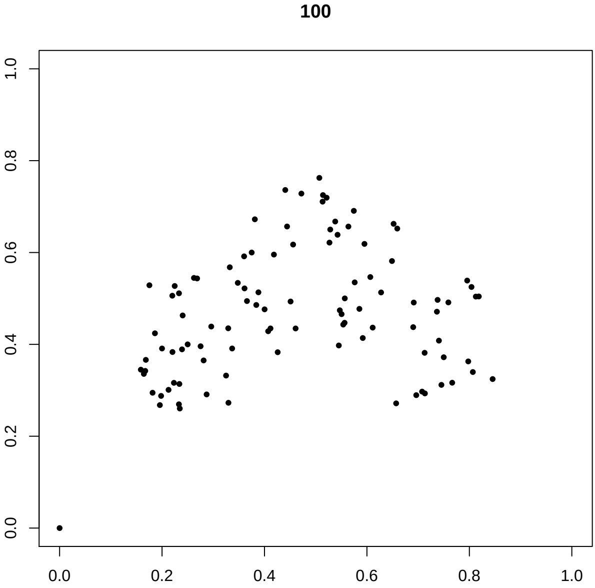

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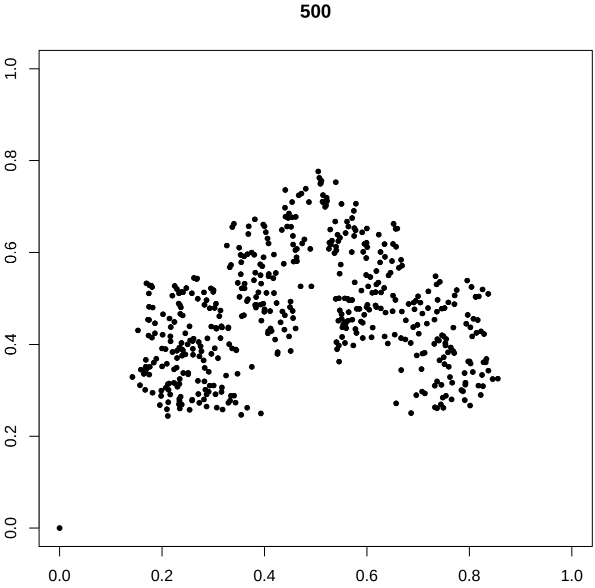

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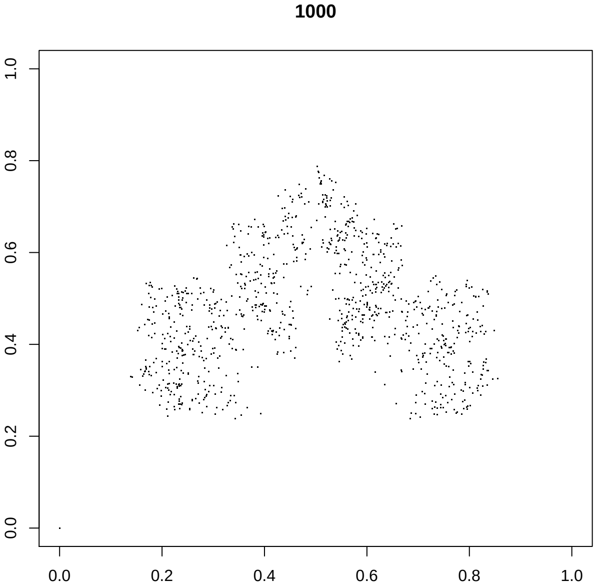

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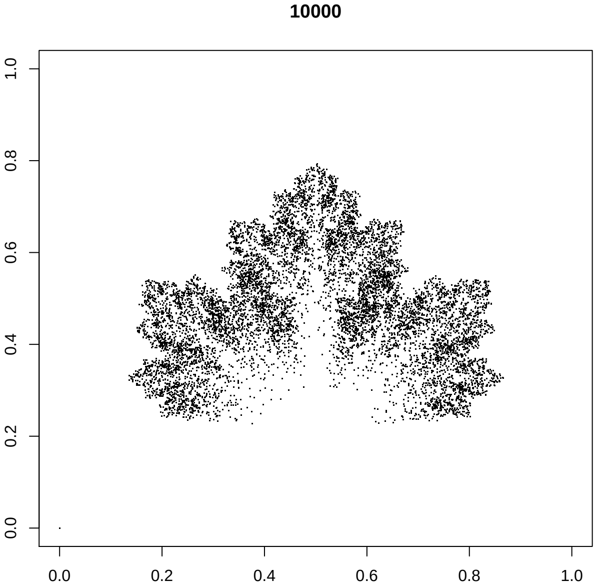

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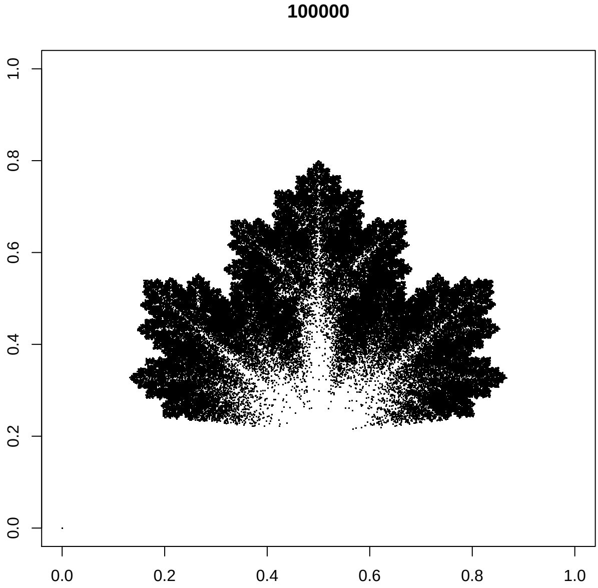
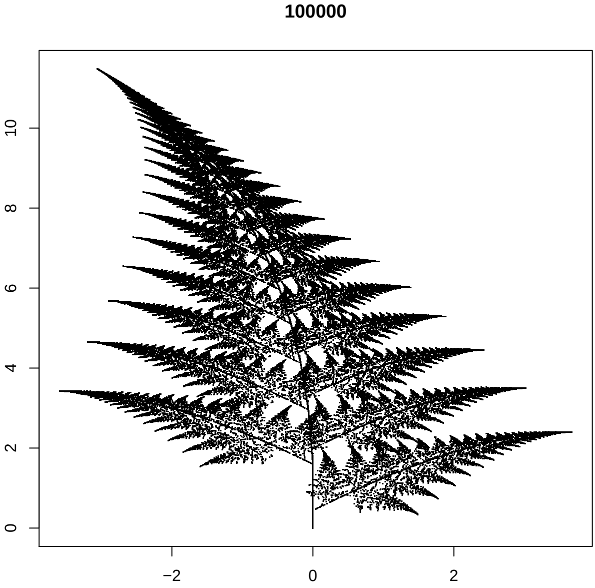


Try it out yourself!

https://arnab-chakraborty.shinyapps.io/shny/

Another strange thing!



Statistical regularity

Regularity in randomness!

- **Not always**
- **Only when we have “lots of randomness”**

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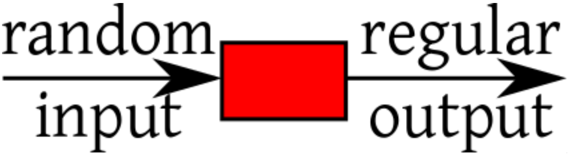
Natural phenomena:

- **Leaves: very similar but not same**
- **Finger prints**

Why care?



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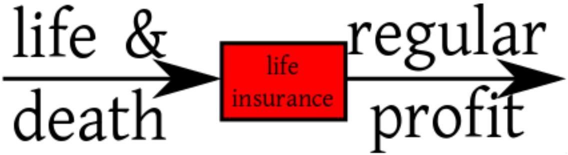

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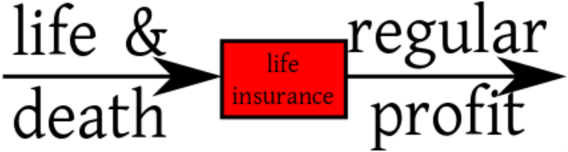
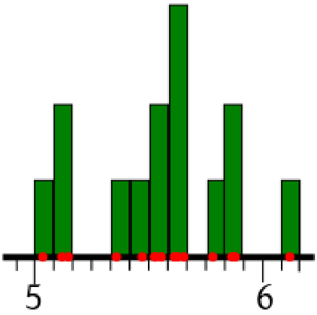


1. **Understanding: Probability**
2. **Using: Statistics**

Histogram

Histogram

Histogram



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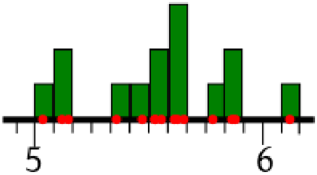
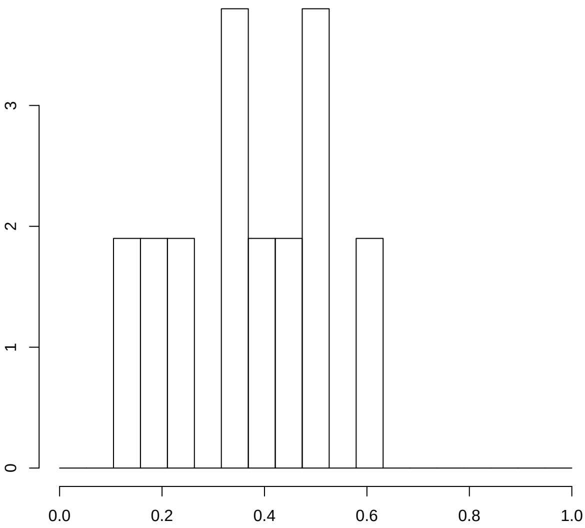


Probability density function



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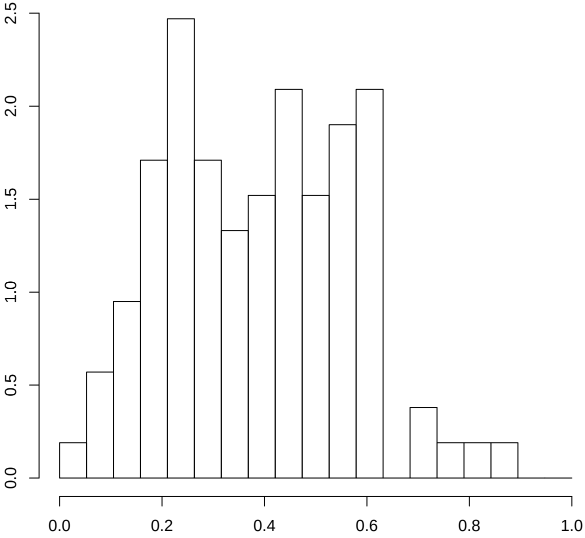

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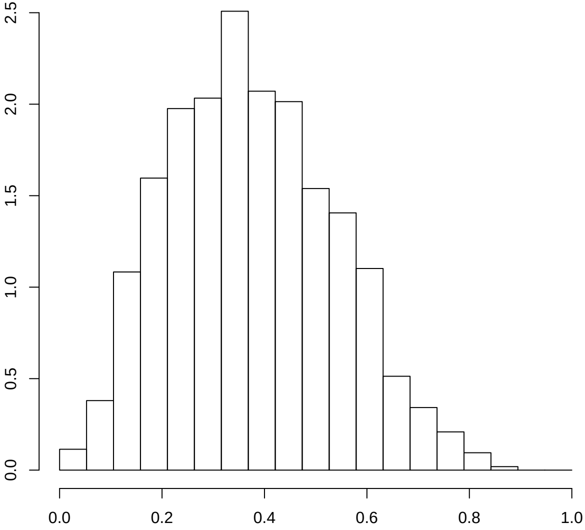

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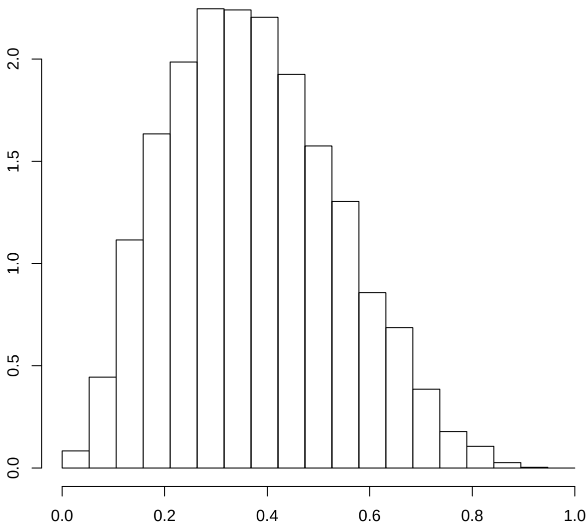

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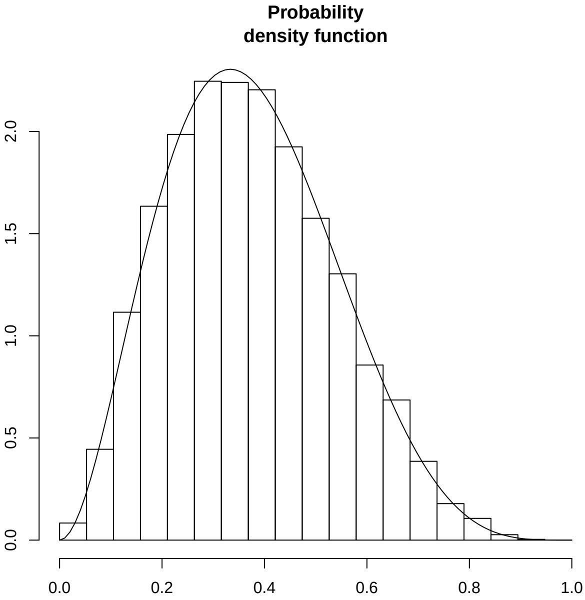

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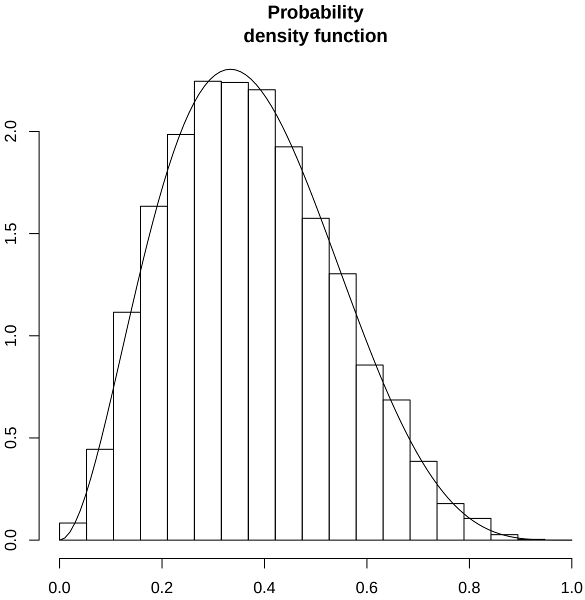

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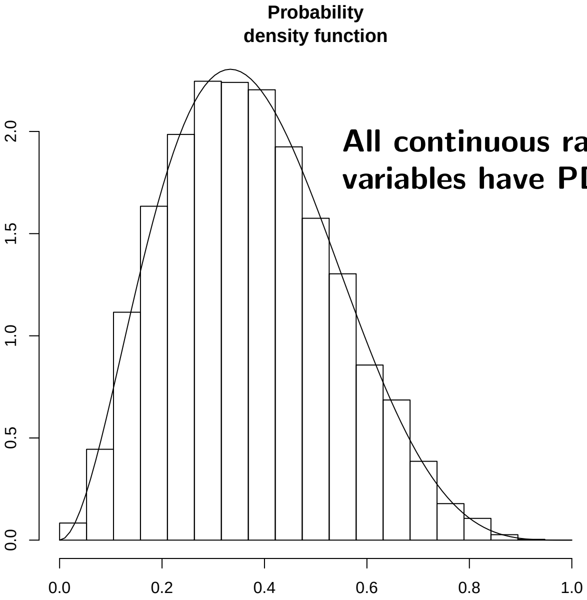
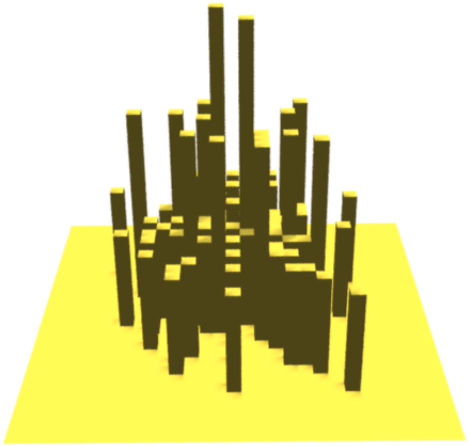


All continuous random variables have PDFs.

Probability density function



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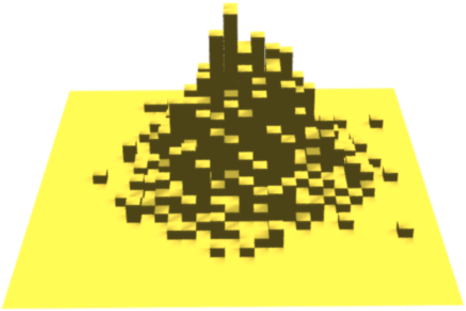

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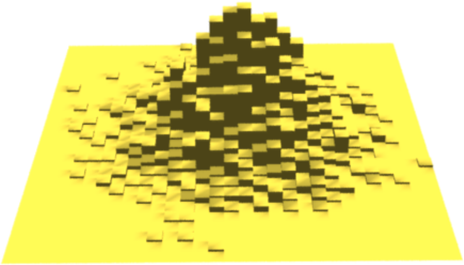

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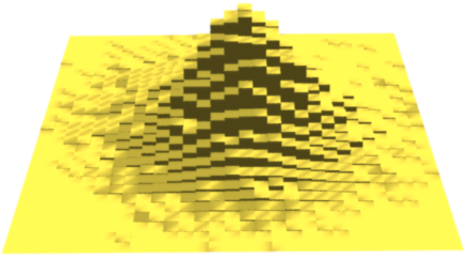

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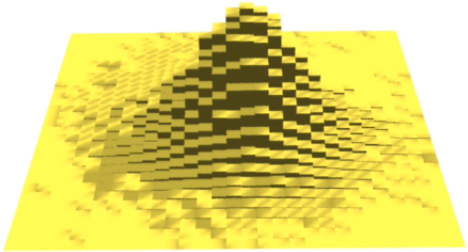

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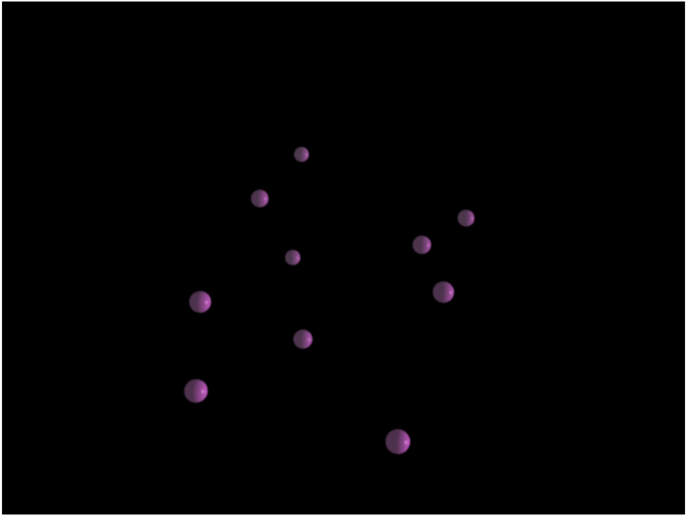
All continuous random variable pairs have joint PDFs.

Molecules



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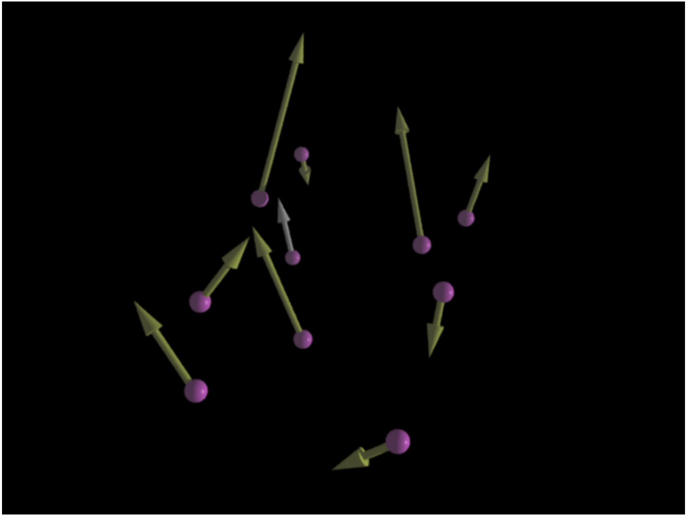
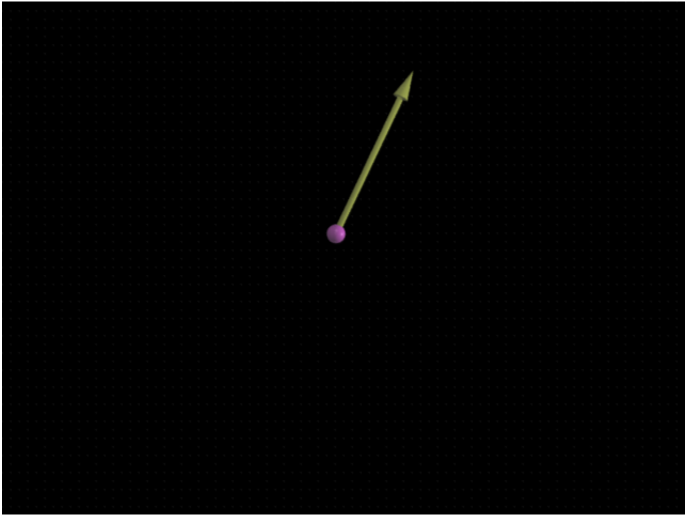


A single molecule



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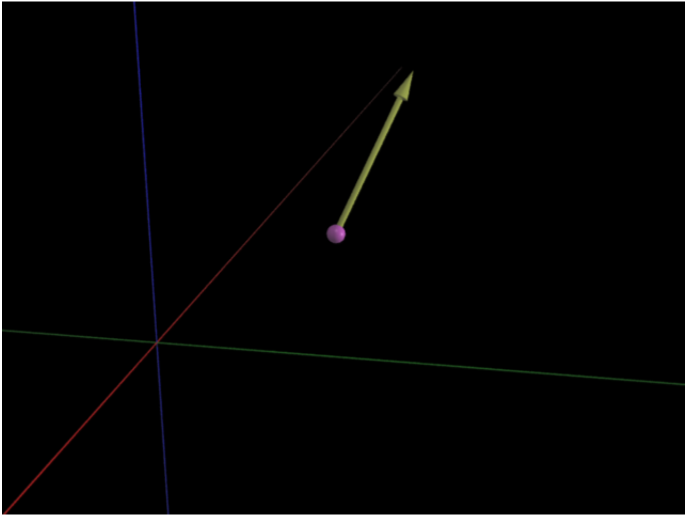

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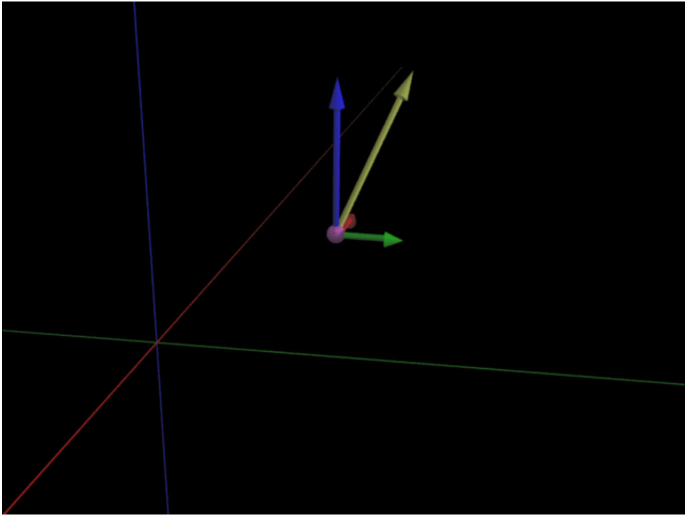

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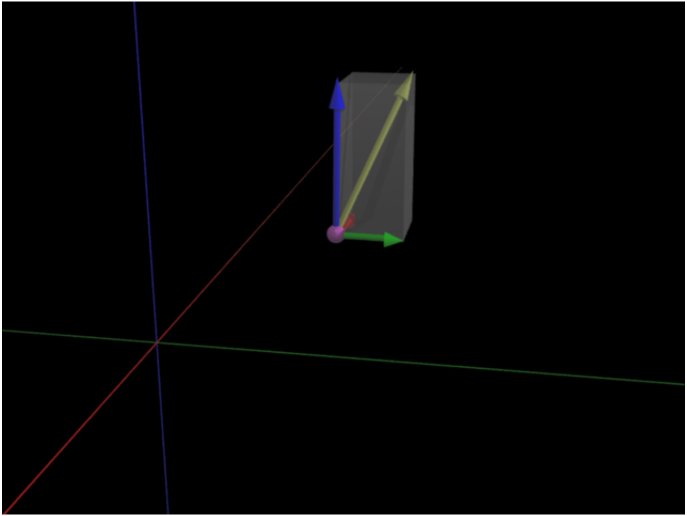

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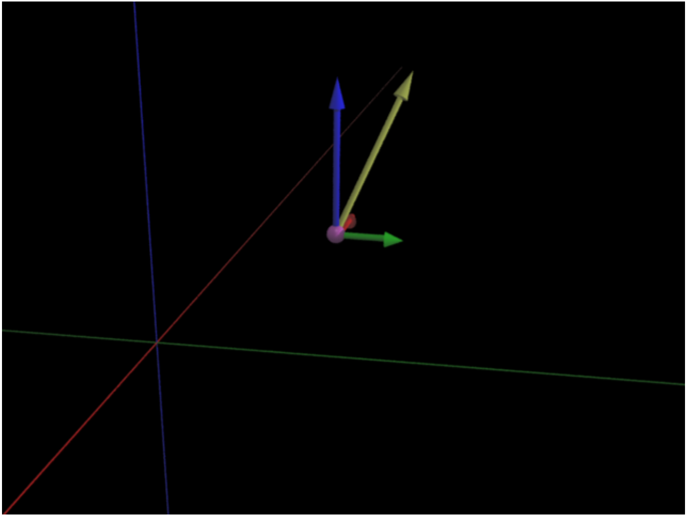

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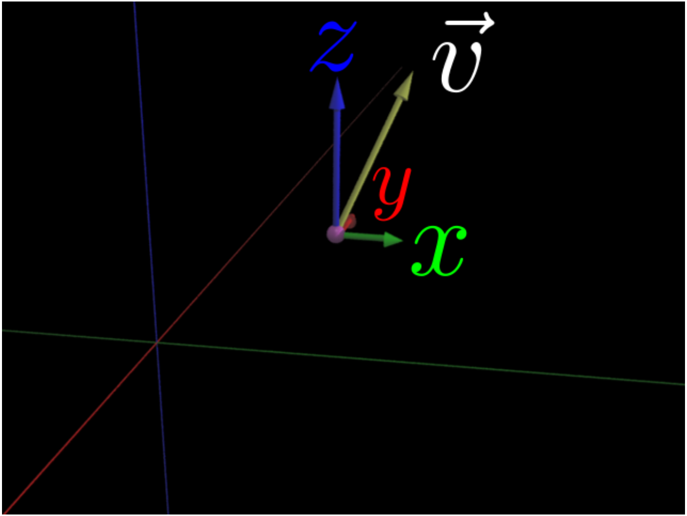

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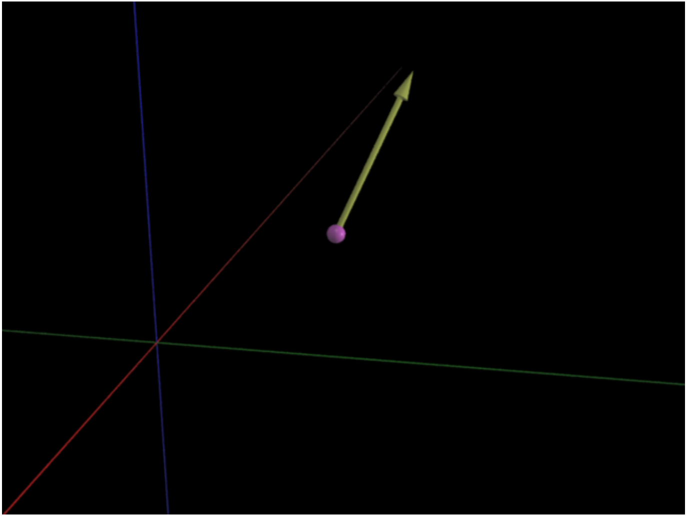
- **So joint density of** x, y, z **is** $f(x)f(y)f(z)$.
- $f(x)f(y)f(z)$ **does not depends only on the length of** (x, y, z) **and not on the direction.**

Isotropy



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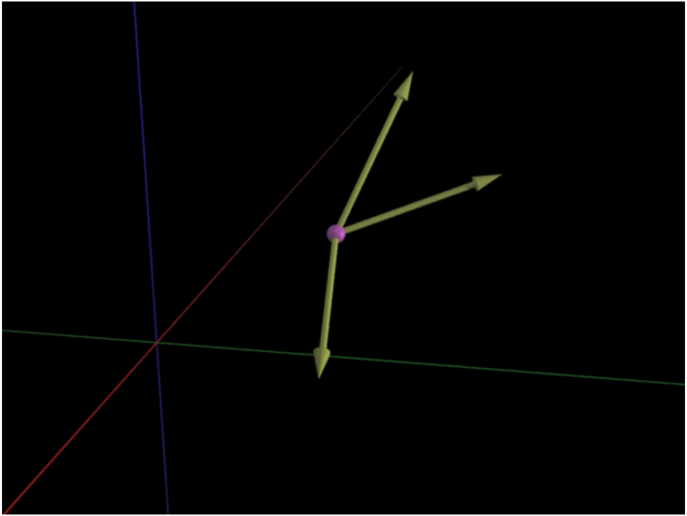

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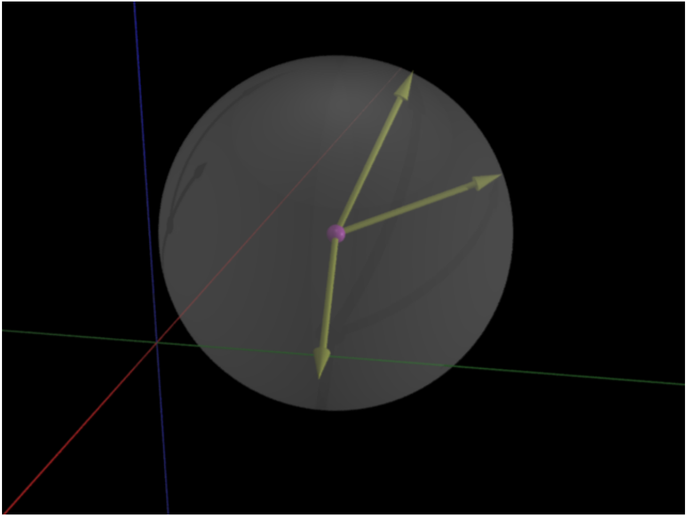

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$$\frac{f'(x)}{xf(x)} = \frac{f'(y)}{yf(y)} = \frac{f'(z)}{zf(z)} = k, \ \textbf{say}.$$

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Maxwell / Gaussian distribution.